NONSTATIONARY CONVECTIVE HEAT TRANSFER WITH EVAPORATION OF A LIQUID FROM THIN-WALLED CAPILLARY-POROUS BODIES UNDER CONDITIONS OF NATURAL CONVECTION

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Nonstationary convective heat transfer accompanying evaporation of water from porous metalloceramic bodies is analyzed. It is established that for low-intensity evaporation of moisture from thin capillary-porous bodies, the bulk evaporation model agrees well with experiment.

Transpiration cooling is currently widely used in various fields of engineering. To ensure reliable and efficient operation of a cooling system, it is necessary to know its working parameters and their variation as a function of the heat- and mass-transfer characteristics. The characteristics of this process in the stationary state have been studied quite well [1]. The problems of nonstationary convective heat transfer in transpiration cooling have not been adequately studied. Nonstationary heat transfer occurs in transpiration cooling systems with cyclical input of water into the porous element, as well as at the initial and final periods of their operation. These problems have been examined mainly for the process of drying of capillary-porous materials [2, 3]. In so doing, it is noted that the nature of the drying process is determined by the method and intensity of energy input to the material and its thermophysical, structural, and geometrical parameters. External heat transfer from moist porous bodies as they dry has been studied primarily for forced convective and radiative methods for introducing energy into the material. For the conductive method, under conditions of natural convection and especially at low pressures, nonstationary convective heat transfer in transpiration cooling has practically not been examined.

In what follows, we investigate the effect of the moisture content of the capillary-porous body on external convective heat transfer with conductive energy input for different pressures of the surrounding medium and for different thermal loads. In [3], conductive drying of porous bodies is examined when the evaporation of moisture occurs from all sides of the drying material. We, however, investigate the removal of liquid from a metalloceramic porous plate as applied to transpiration cooling, when evaporation of moisture occurs only from the external side of the plate, while the internal surface remains sealed from the surrounding medium and is in contact with the surface of the heater (Fig. 1).

A titanium metalloceramic plate ($\delta = 3.2 \cdot 10^{-3}$ m; D = 0.13 m; d = $5 \cdot 10^{-6}$ m; $\Pi = 0.3$) was placed in a thermally insulated metallic housing. A thin flat electric heater with the same shape and size as the porous plate was mounted in the backing. The experimental model of the porous evaporator was placed on scales in a vacuum chamber. The fluid (distilled water) entered along distributing channels beneath the plate and saturated it uniformly to maximum moisture content U_{max}. The experiment consisted of several series, corresponding to different heat flows from the heater ($q_{ex} = (0.5\text{–}6) \cdot 10^3$ W/m^2) and pressures of the surrounding medium ($P = (0.038\text{–}10) \cdot 10^4$ N/m^2). Prior to each series of measurements, a stationary heat-transfer regime was attained by repeated saturation of the porous plate with the liquid. The series of measurements consisted of synchronously measuring the indications of the scales, the heat flux sensors, and the copper–constantan thermocouples, affixed to the inner and outer surfaces of the porous plate, as well as in the surrounding medium far from the plate, from the time of maximum saturation of the plate to its complete drying. The error in determining the temperature did not exceed 6 %.

Figure 2 shows some of the experimental results. As is evident from the figure, the time dependences of the temperature and moisture content of the porous plate are analogous to the corresponding dependences presented in [3]. Here the moisture content is taken to mean the ratio of the quantity of liquid contained in the plate to the maximum quantity of liquid with which it is possible to saturate the plate. As in [3], periods of constant and decreasing rates of evaporation are observed ($U_{cr} = 20\%$). For the same heater power, as the

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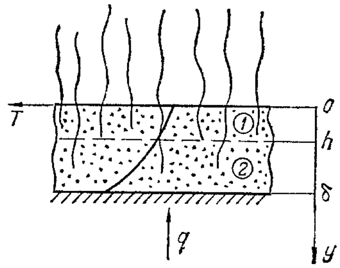

Fig. 1

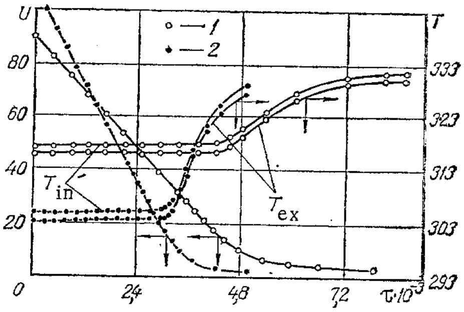

Fig. 2

Fig. 1. Formulation of the problem of nonstationary transpiration cooling of a porous plate.

Fig. 2. Dependence of the moisture content U (%) and temperature T (°K) of a plate on the time τ (sec) during nonstationary transpiration cooling under different pressures of the surrounding medium P (N/m²): 1) $1 \cdot 10^5$; 2) $8 \cdot 10^3$ and the power of the heater is Q = 14.8 (W).

pressure of the medium in the chamber decreases, the rate of increase in temperature and the drop in the rate of evaporation increase due to the increase in heat flux going into evaporation of the liquid. In addition, for moisture content U = 20-5%, the temperature differential on the plate decreases, which is apparently due to the change in the process of evaporation of moisture from the porous plate during this time (onset of the bulk evaporation regime over the entire thickness of the plate).

The coefficients of convective heat transfer of the external surface of the plate to the medium were determined from the energy balance equation for the plate

$$q = rj + \frac{C}{S}\frac{d\bar{T}}{d\tau} + \alpha_{con}(T_{ex} - T_\infty) + q_{rad}, \tag{1}$$

where all three known forms of heat transfer are included. Figure 3 shows the experimental dependences of α_{con} on U for the porous plate studied (curves 1, 2), obtained from data in Fig. 2. The error in determining α_{con} in this case did not exceed 15%. The curves of α_{con}(U) obtained are analogous to the corresponding curves in [2]. Therefore, the introduction of energy into the thin-walled porous body does not affect the form of the function α_{con}(U).

An attempt is made in [4] to apply the generalized theory of the steady-state thermal regime [5] to calculate α_{con} for the process of drying moist bodies. This theory is not applicable in our case (conductive energy input). As demonstrated in [5], the generalized steady-state theory can be used only for heating (cooling) of bodies that contain sources (sinks) of heat with constant power. In a period of decreasing rate of drying, the temperature of the body increases and the rate of evaporation decreases. This means that the magnitude of the heat flux going into heating the body is variable (increases) due to the variable (decreasing) rate of evaporation during this period. Thus, the power of the internal heat source is a variable quantity, which leads to a time-dependent rate of heating of the body. Therefore, the steady-state theory is not applicable in our case.

It is well known [3] that two different drying regimes can be realized depending on the method for introducing energy, the magnitudes of the evaporation rate, thickness of the dried material, and its thermophysical and structural parameters: bulk evaporation moisture and a regime accompanied by recession of the evaporation zone into the body. The first regime occurs with low-intensity drying of thin finely dispersed materials, especially with uhf energy input. The second regime is characteristic of high intensity drying of thin-walled bodies, especially with convective and conductive methods of energy input.

Evaporation of liquid coolant in the porous structure greatly affects the reliability and efficiency of the process of transpiration cooling. It is known, for example, that the regime in which the phase transition surface recedes into a homogeneous porous wall results in instability of the transpiration cooling process when water is used as the coolant [6]. The problem of the stability of this process has now been solved by increasing the hydraulic resistance of the wall (use of a two-layer porous structure) [7]. However, this problem can apparently be solved in a simpler manner by eliminating the reason for the instability: recession of the evaporation surface into the porous wall. This can be achieved by realizing bulk evaporation of coolant from the porous structure.

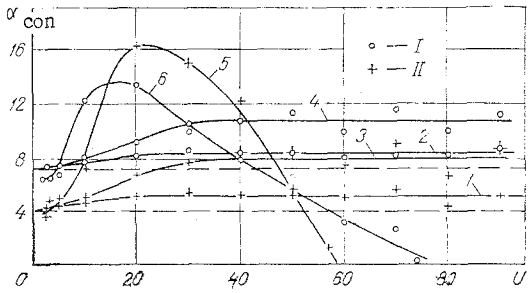

Fig. 3. Dependence of the coefficient of convective heat transfer α_{con}(W/m²·K) of the external surface of a moist porous plate on the moisture content U(%) for different pressures of the surrounding medium P (N/m²): I) $1\cdot10^5$; II) $8\cdot10^3$ and heater power Q = 14.8 (W): 1, 2) experiment; 3, 4) calculation according to the bulk evaporation model; 5, 6) calculation using the model of a recessed evaporation surface.

To clarify the true physical picture of evaporation of fluid from a porous plate, we shall examine two limiting computational models of the process, describing both of the regimes noted above, in application to our problem. It is assumed that the temperatures of the liquid (vapor) and the framework of the plate at any point of the plate volume are equal and that the time derivative of the temperature of the plate is independent of the coordinate y for thin bodies (see Fig. 1).

The first computational model of bulk evaporation of fluid from a plate in our case (Fig. 1) is described by the following differential equation of heat balance

$$c\rho\frac{\partial T}{\partial\tau}=\lambda\frac{\partial^2 T}{\partial y^2}+\frac{rm_{\max}}{V}\frac{dU}{d\tau}-j_v c_v\frac{\partial T}{\partial y},\qquad(2)$$

where

$$c=\frac{C}{\rho V};\ C=c_f m_f+c_l m_l+c_v m_v;\ \rho=\frac{m_f+m_l+m_v}{V};\ V=\delta S.\qquad(3)$$

Writing the starting equation in the form (2), we thereby assume that the rate of decrease of moisture m_{max} ($dU/d\tau$) at any time is independent of the coordinate y. This assumption for this problem is physically completely justified since the heat flux q is oriented in the same direction as the vapor flux (Fig. 1). Thermodiffusion of moisture in a thin plate can be neglected since the temperature differential over the plate is insignificant. Redistribution of moisture within the plate due to capillary forces during drying is neglected. Therefore, the phase transformation criterion $\varepsilon = 1$ [3]. The magnitude of the specific flow rate of vapor j_v, entering into Eq. (2), depends not only on the time τ, but also on the coordinate y. It is not difficult to establish that at any time j_v depends linearly on the coordinate y:

$$j_v=\frac{m_{\max}}{V}\frac{dU}{d\tau}y<0.\qquad(4)$$

Substituting (3) and (4), after transformations, we obtain the starting differential equation for heat transport in the plate for our problem according to the model of bulk evaporation of moisture:

$$\frac{\partial T}{\partial\tau}=\frac{\lambda V}{C}\frac{\partial^2 T}{\partial y^2}+\frac{rm_{\max}}{C}\frac{dU}{d\tau}-\frac{c_v m_{\max}}{C}\frac{dU}{d\tau}y\frac{\partial T}{\partial y}.\qquad(5)$$

The thermophysical parameters of the plate framework and of the fluid and its vapor are assumed to be independent of temperature. The coefficient of thermal conductivity λ of a moist porous plate, depending on its moisture content, is determined experimentally using the technique developed in [8]. Equations of the form (5) are usually solved with the help of the Laplace transformation [9]. The solution thus obtained has a quite cumbersome and complicated form, which is inconvenient for practical applications. For this reason, we shall solve Eq. (5) for the plate under study using a simpler computational-experimental method, which consists of assuming that the time dependent parameters entering into (5) are constant quantities at all times and are taken from experiment (Fig. 2). Thus the solution of Eq. (5) for each time reduces to a quasistationary problem, which is described by a linear inhomogeneous equation of the form

$$\frac{d^2T}{dy^2}-By\frac{dT}{dy}=A,\qquad(6)$$

where

$$B=B(\tau)=\frac{c_v m_{\max}}{\lambda V}\frac{dU}{d\tau}<0;\ A=A(\tau)=\frac{C\dfrac{\partial T}{\partial\tau}-rm_{\max}\dfrac{dU}{d\tau}}{\lambda V}>0.$$

The parameters A and B at each time are determined from experiment. The boundary conditions of the problem are boundary conditions of the first kind:

$$y = 0, \quad T = T_v, \quad y = \delta, \quad T = T_{in}. \tag{7}$$

The general solution of Eq. (6), obtained with the help of the method of variation of arbitrary constants, is cumbersome and inconvenient to use:

$$T = \int \left[A \int \left(\frac{d\varphi}{dy} \right)^{-1} dy \right] d\varphi + K + \varphi L; \tag{8}$$

where $\varphi = \sqrt{-\pi/2B}\ \mathrm{erf}\,(y\sqrt{-B/2})$. This makes it necessary to use numerical methods.

We shall simplify the solution of Eq. (6). The corresponding homogeneous equation has the form

$$\frac{d^2\Theta}{dy^2} - By \frac{d\Theta}{dy} = 0. \tag{9}$$

We shall present the general solution of Eq. (9):

$$\Theta = L_0 \int \exp\left(\frac{B}{2} y^2 \right) dy + K_0. \tag{10}$$

We expand $\exp\left(\frac{B}{2} y^2\right)$ in a Taylor series and retain the first two terms in the series. This simplification, even for maximum rates of evaporation in drying capillary-porous bodies, gives an error that does not exceed 0.2%. As a result of the simplification made, after appropriate operations, we obtain the general solution of Eq. (6) in analytic form:

$$T = A \sqrt{-\frac{1}{2B}} \left(y + \frac{B}{6} y^3 \right) \ln \left(\frac{1 + y\sqrt{-\frac{B}{2}}}{1 - y\sqrt{-\frac{B}{2}}} \right) - A \left(\frac{y^2}{6} + \frac{2}{3} \frac{\ln \left| 1 + \frac{B}{2} y^2 \right|}{B} \right) + L \left(y + \frac{B}{6} y^3 \right) + K. \tag{11}$$

The integration constants K and L are determined from the boundary conditions (7), which at each time are known from experiment. The heat balance condition is satisfied on the external surface of the porous plate:

$$\lambda \left(\frac{dT}{dy} \right)_{y=0} = \alpha_{con}(T_{ex} - T_\infty) + q_{rad}. \tag{12}$$

We then write the coefficient of external convective heat transfer for each time as

$$\alpha_{con} = \frac{\lambda \left(\frac{dT}{dy} \right)_{y=0} - q_{rad}}{T_{ex} - T_\infty} = \frac{\lambda L - q_{rad}}{T_{ex} - T_\infty}. \tag{13}$$

It should be noted that this model for calculating α_{con} is further simplified if convective heat transfer by vapor is neglected. This assumption is completely justified for thin-walled capillary-porous bodies. In this case, the last term on the right side of Eq. (5) is eliminated. Then the problem reduces to solving the linear inhomogeneous equation

$$\frac{d^2T}{dy^2} = A. \tag{14}$$

Its general solution has the form

$$T = \frac{A}{2} y^2 + Ky + L, \tag{15}$$

where K and L are determined from (7):

$$L = T_{ex}, \quad K = \frac{T_{in} - T_{ex} - A\delta^2/2}{\delta}. \tag{16}$$

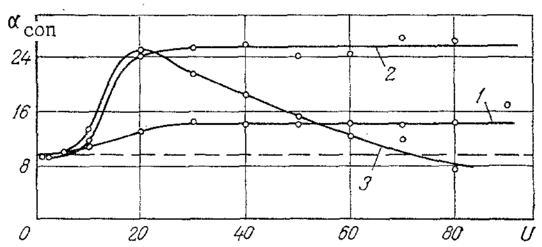

Fig. 4. Dependence of the coefficient of convective heat transfer α_{con}(W/m^2·K) of the external surface of a moist porous plate on the moisture content U(%) with the surrounding medium at atmospheric pressure and heater power Q = 30 (W): 1) experiment; 2) calculation using the bulk evaporation model; 3) calculation using the model with recessed evaporation surface.

The expression for α_{con} in this case is written as follows:

$$\alpha_{con} = \frac{\lambda (T_{in} - T_{ex} - A\delta^2/2) - \delta q_{rad}}{\delta (T_{ex} - T_\infty)} . \tag{17}$$

A calculation to verify the quantity α_{con} from expressions (13) and (17) for the porous plate under study confirmed the validity of the assumption made above. Using the experimental results (Fig. 2), the values of α_{con} were calculated using expression (17) (curves 3 and 4 in Fig. 3). As is evident from Fig. 3, the nature of the dependence α_{con}(U), obtained for the working model of bulk evaporation, is analogous to the nature of the experimental dependence α_{con}(U), but the computed values of α_{con} are somewhat greater than the corresponding experimental values.

The second limiting working model of evaporation of fluid from a porous plate, which accounts for the recession of the surface of evaporation (see Fig. 1), is analogous to Stefan's problem [10] in our case and it is described by two differential equations for heat transfer:

region 1

$$h \geqslant y \geqslant 0, \quad \rho_1 c_1 \frac{\partial \bar{T}}{\partial \tau} = \lambda_1 \frac{\partial^2 T_1}{\partial y^2} - \frac{m_{max} c_v}{S} \frac{dU}{d\tau} \frac{\partial T_1}{\partial y} ; \tag{18}$$

region 2

$$\delta \geqslant y \geqslant h, \quad \rho_2 c_2 \frac{\partial \bar{T}}{\partial \tau} = \lambda_2 \frac{\partial^2 T_2}{\partial y^2} , \tag{19}$$

which are solved by the computational–experimental method described above and after transformations are put into the form

$$0 \leqslant y \leqslant h, \quad \frac{d^2 T_1}{dy^2} - B_1 \frac{dT_1}{dy} = A_1, \tag{20}$$

$$h \leqslant y \leqslant \delta, \quad \frac{d^2 T_2}{dy^2} = A_2, \tag{21}$$

where

$$A_1 = A_1(\tau) = \frac{C_1 \partial \bar{T}/\partial \tau}{\lambda_1 V_1} > 0, \quad B_1 = B_1(\tau) = \frac{c_v h m_{max}}{\lambda_1 V_1} \frac{dU}{d\tau} < 0,$$

$$A_2 = A_2(\tau) = \frac{C_2 \partial \bar{T}/\partial \tau}{\lambda_2 V_2} > 0, \; C_1 = c_f m_{f_1} + c_v m_v, \; C_2 = c_f m_{f_2} + c_l m_l ,$$

$$V_1 = hS, \; V_2 = (\delta - h) S, \; h = \delta \frac{U_{max} - U}{U_{max}} .$$

At each time, the parameters A_1, B_1, A_2 are constant quantities and are determined from experiment. The boundary conditions from (20) and (21) are as follows:

$$0 \leqslant y \leqslant h, \quad \begin{cases} y = 0, & T_1 = T_{ex}, \\ y = h, & T_1 = T_{se}; \end{cases} \tag{22}$$

$$h \leqslant y \leqslant \delta, \quad \begin{cases} y = h, & T_2 = T_{se}, \\ y = \delta, & T_2 = T_{in}. \end{cases} \tag{23}$$

The general solution of Eqs. (20) and (21) has the form

$$T_1 = -\frac{A_1}{B_1}\left(y + \frac{1}{B_1}\right) + K_1 \exp(B_1 y) + L_1, \tag{24}$$

$$T_2 = \frac{A_2}{2} y^2 + K_2 y + L_2. \tag{25}$$

The integration constants K_1, L_1, K_2, L_2 are determined from the boundary conditions (22) and (23) and include the unknown temperature of the surface of evaporation T_{se} within the plate at each time. The quantity T_{se} is determined from the boundary condition of the fourth kind with y = h:

$$\lambda_2 \left(\frac{dT_2}{dy}\right)_{y=h} = \lambda_1 \left(\frac{dT_1}{dy}\right)_{y=h} + rj, \tag{26}$$

where

$$j = \frac{m_{\max}}{S} \left|\frac{dU}{d\tau}\right| > 0.$$

After transformations, we obtain from (26) using (24) and (25) an expression for T_{se}:

$$T_{se} = \frac{\left\{\left[\frac{A_2}{2}\lambda_2 (h - \delta) + \frac{A_1}{B_1}\lambda_1 - rj\right](\delta - h) + \lambda_2 T_{in}\right\}[\exp(B_1 h) - 1] + \lambda_1 B_1 \left(T_{ex} - \frac{A_1}{B_1} h\right)(\delta - h)\exp(B_1 h)}{\lambda_2 [\exp(B_1 h) - 1] + \lambda_1 B_1 (\delta - h)\exp(B_1 h)}. \tag{27}$$

Knowing the temperature T_{se} for each time, it is possible to find all the parameters required to determine α_{con} according to the working model:

$$\alpha_{con} = \frac{\lambda_1 \left(\frac{dT_1}{dy}\right)_{y=0} - q_{rad}}{T_{ex} - T_\infty} = \frac{\lambda_1 (K_1 B_1 - A_1/B_1) - q_{rad}}{T_{ex} - T_\infty}. \tag{28}$$

We also note that as in the working model of bulk evaporation, for thin-walled porous bodies, convective heat transfer by vapor can be neglected, which simplifies the solution of Eq. (18). The results of the calculation of the quantity α_{con} using the model in which the surface of evaporation is recessed are presented in Fig. 3 (curves 5, 6). As can be seen from the figure, the nature of the function $\alpha_{con}(U)$ for the working model with a recessed surface is analogous to the results obtained experimentally in [11]. Figure 4 presents the experimental and computed dependences α_{con} (U) for higher thermal loads from the heater with the surrounding medium at atmospheric pressure. The dashed lines in Figs. 3 and 4 indicate the value of the coefficient α_{con} for a dry plate.

Comparing the experimental dependences $\alpha_{con}(U)$ with the corresponding dependences obtained from the two limiting working models of evaporation, it may be concluded that the working model of bulk evaporation of fluid for a plate gives results that are closer to the experimental results than the model of evaporation with the recessed phase-transition surface. The incomplete correspondence of the computational results using the bulk evaporation model with the experimental results, which is mainly observed for $U \geq U_{cr}$, is apparently explained in part by the experimental error as well as by the presence of moisture redistribution in the pores due to capillary forces, which is accompanied by recession of the evaporation surface in macrocapillaries as a result of fluid pulled out of them into microcapillaries.

For a higher thermal load (rate of evaporation) (Fig. 4), the disagreement between the experimental results and calculations using the bulk evaporation model increases, while the disagreement with the calculations using the model with a recessed evaporation surface decreases. Nevertheless, the nature of the dependence $\alpha_{con}(U)$, obtained from the working model of bulk evaporation, in contrast to the model with a recessed evaporation surface, remains the same as in the experiment. For $U \leq 5\%$, the experimental results and the computational results obtained using both models practically coincide and correspond to "dry" heat transfer. Thus the computational bulk evaporation model satisfactorily corresponds to the real drying process in the given metalloceramic plate. We can expect that this correspondence will improve with decreasing plate thickness, rate of evaporation of moisture out of the plate, and average pore size.

The computational–experimental method presented in this paper permits automating the calculation of convective heat transfer from experimental data with the help of appropriate means for automating the performance and analysis of the experiment. In so doing, it is necessary that the working model adopted describe sufficiently accurately the real process of evaporation of fluid from the porous body.

NOTATION

α_{con}, coefficient of convective heat transfer; δ, D, and d, thickness, diameter, and average pore size in the plate, respectively; P, porosity; U, U_{max}, instantaneous and maximum value of the moisture content of the plate, respectively; q, specific heat flux from the heater to the plate; q_{rad}, specific radiant heat flux from the surrounding medium to the plate; T, temperature; $\bar{T}$, average temperature of the plate; S, surface area of the plate; m, mass; ρ, density of the moist porous structure; c, specific heat capacity; j, specific rate of evaporation; τ, time; V, volume of the plate; r, specific heat of evaporation of the liquid; λ, coefficient of thermal conductivity of the plate; m_{max}, maximum amount of liquid with which the plate can be saturated; h, coordinate of the surface of evaporation of the plate; C, bulk heat capacity. The indices are as follows: f, framework; l, liquid; v, vapor; cr, critical; ex, external surface of the plate; in, internal surface of the plate; se, surface of evaporation; 1, region of the first plate; 2, region of the second plate; and ∞, surrounding medium far away from the plate.

LITERATURE CITED

1. V. A. Maiorov and L. L. Vasil'ev, "Heat transfer and stability with motion of a coolant, evaporating in porous metalloceramic materials," Inzh.-Fiz. Zh., 36, No. 5, 914-934 (1979).
2. P. D. Lebedev, Drying by Infrared Rays [in Russian], Gosénergoizdat, Moscow (1955).
3. A. V. Lykov, Theory of Drying [in Russian], Énergiya, Moscow (1968).
4. A. I. Chaplina, "Criterion for steady-state thermal regime under different conditions for heating bodies," Inzh.-Fiz. Zh., 5, No. 9, 118-125 (1962).
5. G. N. Dul'nev and G. M. Kondrat'ev, "Generalized theory of steady-state thermal regime," Izd. Akad. Nauk SSSR, Otd. Tekh. Nauk, No. 7, 71-85 (1956).
6. A. V. Luikov, L. L. Vasiliev, and V. A. Mayorov, "Determination of the region of stable and reliable operation of equilibrium two-phase transpiration cooling system," Int. J. Heat Mass Transfer, 18, No. 7/8, 885-892 (1975).
7. V. A. Maiorov, "Development of stable system of two-phase transpiration cooling," Inzh.-Fiz. Zh., 32, No. 5, 870-877 (1977).
8. P. A. Novikov and B. G. Mikhnyuk, "Thermal conductivity of porous materials as a function of moisture content," Inzh.-Fiz. Zh., 21, No. 2, 296-300 (1971).
9. A. V. Lykov, Theory of Heat Conduction [in Russian], Vysshaya Shkola, Moscow (1967).
10. H. S. Carslaw and J. C. Jaeger, Conduction of Heat in Solids, Oxford Univ. Press (1959).
11. G. Puasson, "Heat exchange between flow of rarefied air and a porous body saturated with water," in: Heat and Mass Transfer [in Russian], Vol. 1, Pt. 3, Nauka i Tekhnika, Minsk (1971), pp. 209-221.